AN IMPROVED SOLUTION TO ESTIMATE THE STRESS STATE IN SUB-VERTICAL BACKFILLED STOPES

Li Li and Michel Aubertin
Department of Civil, Geological and Mining Engineering – École Polytechnique, Montreal, Quebec, Canada

ABSTRACT

The increasing use of backfill in underground mines requires a better understanding of the interaction between the relatively soft fill material and the surrounding rock mass. In recent years, it has been shown that stresses in backfilled stopes can be estimated using an approach based on Marston’s arching formulation developed initially for buried conduits in trenches. However, despite its advantages, this approach has some shortcomings. For instance, it postulates that both the vertical and horizontal stresses are uniformly distributed across the opening width. Numerical investigations conducted by the authors have shown that this assumption is not always valid. This paper presents a modification to the Marston-based solution, which leads to a non-uniform vertical stress distribution across the opening. This modification of the analytical solution involves parameters that have been calibrated against some numerical modelling results obtained with FLAC-2D.

RÉSUMÉ

L'accroissement de l'utilisation des remblais dans les mines souterraines requiert une meilleure compréhension de l'interaction entre le matériau de remblayage, relativement déformable, et le massif rocheux encadrant. Au cours des dernières années, il a été démontré que les contraintes dans les chantiers remblayés peuvent être estimées en utilisant une approche basée sur la solution de Marston, initialement développée pour des conduites en tranchées. Mais, malgré ses avantages, cette approche souffre de certaines limitations. Par exemple, elle postule que les contraintes verticale et horizontale sont uniformes sur toute la largeur de l'ouverture. Des investigations numériques menées par les auteurs ont révélé que cette hypothèse n'est pas toujours valide. Dans cet article, on présente une modification de la solution basée sur l'approche de Marston. Elle implique une distribution non-uniforme des contraintes verticales sur la largeur du chantier. Cette modification de la solution analytique inclut des paramètres calibrés selon des résultats numériques obtenus avec FLAC-2D.

1 INTRODUCTION

Backfilling has become a standard practice in mining operations. One of the main reasons behind the extensive use of backfill in mines is that it can lead to a significant reduction of the amount of wastes disposed on the surface. Backfilling thus protects the landscape and reduces the potential for environmental impacts such as acid mining drainage (when sulphide minerals are present). Despite this advantage, it must be kept in mind that the main role of mine backfill is to provide ground support to ensure stope stability and improve ore recovery by reducing dilution (e.g., Hassani and Archibald 1998; Kump 2001; Jung and Biswas 2002). It is thus important for ground control engineers to have well adapted tools that allow a relatively fast and reliable estimate of the stress state in and around backfilled stopes. In this regard, it must be recalled that most backfill materials, even those that include a significant portion of binder, are relatively soft and show a low yielding strength compared to the surrounding rock mass (e.g., Belen et al. 2000, 2002; Benzaazoua et al. 2002). As a consequence, the backfill placed in a stope tends to settle significantly, often producing fairly large strains (of 5% or more; e.g., Belem et al. 2007). The deformation of the backfill may then induce some load transfer along the interfaces due to the frictional forces that develop along the contact with the rock mass (e.g., Aubertin et al. 2003). This type of phenomenon, often encountered with confined backfills, is known as arching (e.g., Handy 1985; Hunt 1986; Harrop-Williams 1989; Drescher 1991).

Arching effects have been observed in many situations, including within grain silos and powder bins (e.g., Cowin 1977; Blight 1986a, b). Basic arching theory (first developed by Janssen, 1895) has been used to develop solutions for these situations and for other problems encountered in geotechnique. One of the pioneering studies in this regard is that of Marston (1930) and collaborators who used this approach to evaluate vertical loads on conduits placed in trenches (e.g., Spangler and Handy 1984; McCarthy 1988). Terzaghi (1943) also used a somewhat similar approach to assess the stress state above horizontal circular openings (e.g., Ladanyi and Hoyaux 1969; Iglesia et al. 1999). Other applications of arching theory have also been developed for retaining walls (Take and Valsangkar 2001) and dams with confined cores (Kutzner 1997). An important feature of
basic arching theory involves a transfer mechanism which typically reduces stress in the backfill as the vertical loads are transferred to the surrounding stiffer abutments. Nevertheless, stress increases due to a reverse (negative) arching effect may occur when the fill is surrounded by a softer material (e.g., McCarthy 1988; Brachman and Krushelnitzky 2005).

The most versatile means for studying the complex interaction between backfill and the surrounding walls involves numerical modelling tools. These can consider various influence factors such as natural stress conditions, excavation and placement sequence, discontinuities and geometry of the opening (e.g., Pariseau 1981; Hustrulid et al. 1989; Brummer et al. 1996; Brechtel et al. 1999; Aubertin et al. 2003; Li et al. 2003, 2007; Pirapakaran and Sivakugan 2007). In the early stage of a project, however, analytical methods may also provide simple, low cost solutions for a preliminary evaluation of the stress state in backfilled openings (e.g., van Horn 1964; Knutsson 1981; Mitchell 1983; Aubertin 1999; Aubertin et al. 2003; Li et al. 2005a). Previous studies have shown that analytical solutions based on arching theory can be well suited for estimating earth pressures in vertical openings. Such formulations are known, however, to suffer from some limitations due to the simplifying assumptions behind their development. For instance, most existing solutions assume that both the vertical and horizontal stresses are uniformly distributed across the width of the opening. Numerical calculations conducted by Li et al. (2003) have shown, however, that this is not always a valid hypothesis.

In this paper, the analytical solution previously developed by the authors for evaluating the stress state in vertical stopes is modified to account for a non-uniform stress distribution across the stope width. Numerical modelling results are used to define the correction factors applied to the solution, and also for calibrating the controlling parameters. Additional calculations are made to assess the capability of the modified analytical solution.

2 STRESS STATE ACCORDING TO MARSTON’S APPROACH

2.1 Basic Formulation

Figure 1 shows a vertical backfilled opening; $H$ is the backfill height and $B$ is the opening width. At depth $h$, the horizontal layer element is subjected to a lateral compressive force $C$, a shearing force $S$, and the vertical forces $V$ and $V + dV$. The weight of the backfill in this layer is given by:

$$W = \gamma B \, dh$$

where $\gamma$ is the unit weight of the backfill, and $dh$ is the thickness of the layer element. The global equilibrium of the layer element also gives:

$$W = dV + 2S$$

Assuming a cohesionless material with a uniform stress distribution across the width of the opening, the solution of Equation [2] leads to the following equations (Aubertin et al. 2003):

$$\sigma_{vh} = \frac{\gamma B}{2 \tan \delta} \left(1 - \exp \left(-\frac{2 \tan \delta}{h} \right) \right)^{-1}$$

$$\sigma_{bh} = \frac{\gamma B}{2 \tan \delta} \left(1 - \exp \left(-\frac{2 \tan \delta}{h} \right) \right)^{-1}$$

where $\sigma_{vh}$ and $\sigma_{bh}$ are the vertical and horizontal stresses at depth $h$, respectively. Here, $\delta$ is the friction angle of the backfill. The reaction coefficient $K$ (also called earth pressure coefficient) is defined as the ratio of the horizontal stress $\sigma_h$ over the vertical stress $\sigma_v$ ($K = \sigma_h/\sigma_v$). This reaction coefficient depends on the horizontal wall movement and material properties (and on the opening width $B$ in some cases). When the walls do not move, the fill is said to be at rest and the reaction coefficient is typically expressed as (Bowles 1988):

$$K = K_0 = 1 - \sin \phi$$

Cases where an inward movement of the walls compresses the fill would lead to a reaction coefficient comparable to the passive condition $K_p$. This situation is not considered here since previous evaluations have indicated that the stress state is best described by considering that the backfill is close to an active state (e.g., Li et al. 2003, 2005a). Hence, the active reaction coefficient (eq. [6]) is used in the following analytical solutions (i.e. $K = K_a$).

2.2 Modification of the Basic Solution

Li et al. (2003) have shown that the assumption of a uniform stress distribution holds well for $\sigma_v$ across the width of the stope; sample results illustrating this are shown in Figure 2a. On the other hand, the vertical stress is typically not constant across the stope width, as shown in Figure 2b. The analytical solution presented above is thus modified to take into account a non-uniform vertical stress distribution.
Solving this equation (with $\sigma_v = 0$ at $h = 0$), one obtains:

$$\sigma_{v0} = \frac{\gamma B}{2K\delta} \left(1 - \exp \left(\frac{-2K\tan\delta}{B(1-DF)} h\right)\right)$$  \[14\]

The horizontal stress then becomes:

$$\sigma_h = \frac{\gamma B}{2\tan\delta} \left(1 - \exp \left(\frac{-2K\tan\delta}{B(1-DF)} h\right)\right)$$  \[15\]

The vertical stress across the width of the stope is then expressed as:

$$\sigma_{v0} = \frac{\gamma B}{2K\delta} \left(1 - \exp \left(\frac{-2K\tan\delta}{B(1-DF)} h\right)\right) \left[1 - d\left(\frac{x}{B}\right)^\phi\right]$$  \[16\]

As can be seen, Equations 15 and 16 reduce to the basic solution (Eqs. [3] and [4]) when parameter $a = 0$ (or when $b$ is very large).

Figure 3a shows the influence of parameter $a$ on the vertical and horizontal stress distribution along the vertical center-line of the stope. These are plotted with the two stress components based on overburden weight. Figure 3b shows the same stresses along the walls. While the difference between the original and the modified solution is not significant for the center-line, it can become important for the vertical stress component near the wall (Figure 3b). Additional results illustrating the influence of parameter $a$ are also shown in Figure 4. The significant curvature produced by increasing the value of $a$ is seen very clearly in Figure 4a, for the vertical stress.

The influence of parameter $b$ on the stress distribution across the width of the stope is presented in Figure 5. One sees that increasing the value of $b$ tends to flatten the distribution curve. The original Marston solution corresponds to a very large value of $b$ (or to a value of $a = 0$, as shown above).

2.3 Distribution Factor Calibration

It can be expected that the vertical stress distribution across the width is influenced by the stope geometry (width $B$) and by the properties of the backfill (friction angle $\phi$). In order to calibrate the two parameters ($a$ and $b$) that control the distribution factor $DF$, several numerical calculations were conducted with FLAC-2D (Itasca 2002) according to a procedure described in Li et al. (2003), but with 15 backfilling steps. The following results were obtained:

$$a = 2\left(\frac{1}{\beta}\right)\tan^3(\phi_0 + \phi)$$  \[17\]

$$b = 3$$  \[18\]

For the conditions at hand, $\phi_0 = 50^\circ$ and $\lambda = 0.1$ are used for all calculations.

---

Figure 2. Numerical modelling results of the horizontal (a) and vertical (b) stress distribution across the width of a vertical backfilled stope (calculation results with single step backfilling taken from Li et al. 2003).

Numerical results indicate that the vertical stress, $\sigma_v$, can be represented by an equation that follows the shape of an arch, such as:

$$\sigma_v = \sigma_v(0) \left[1 - a\left(\frac{x}{B}\right)^b\right]$$  \[7\]

where $\sigma_v(0)$ is the value of the vertical stress variable at the center of the stope ($x = 0$, $x$ is the distance from the center line; $x \leq B/2$); parameters $a$ and $b$ control the curvature of the stress distribution. In this case, the vertical force acting on the layer element shown in Figure 1 becomes:

$$V = 2\int_0^{B/2} \sigma_v \, dx = B(1-DF)\sigma_v$$  \[8\]

where the distribution factor $DF$ can be expressed:

$$DF = \frac{a}{2^b(b + 1)}$$  \[9\]

The vertical force variation is given by:

$$dV = B(1 - DF)\, d\sigma_v$$  \[10\]

The horizontal stress is considered constant across the width $B$, and proportional to the vertical stress at the center of the stope:

$$\sigma_h = K\sigma_v(0)$$  \[11\]

The shearing force acting on the layer element is determined from the Coulomb criterion:

$$S = \sigma_h \tan\delta \, dh = K\sigma_v(0) \tan\delta \, dh$$  \[12\]

With Eq. [2], this gives:

$$\frac{d\sigma_v}{dh} + \frac{2K\tan\delta}{B(1 - DF)} \sigma_v - \frac{\gamma}{1 - DF} = 0$$  \[13\]
Figure 3. Influence of parameter $a$ value on the vertical and horizontal stress distribution along the center-line (a) and wall (b) of the stope (with $b = 3$).

Figure 4. Influence of parameter $a$ on the vertical (a) and horizontal stress distribution across the width of the stope (with $b = 3$).

Figure 5. Influence of parameter $b$ on the vertical (a) and horizontal (b) stress distribution across the width of the stope (with $a = 3$).

2.4 Sample Comparisons between Analytical and Numerical Results

Additional numerical calculations have been made for different stope widths and backfill properties according to the procedure described by Li et al. (2003). Here, the backfilling is simulated in 15 steps (instead of one step) to
reduce the effect of non-static loading in FLAC (each step corresponds to one fill layer). Typical results are shown in Figures 6 to 8 for a stope width varying from 3 m to 18 m (with \( H = 45 \) m). Figure 6 shows the comparison of vertical and horizontal stresses obtained from the numerical modelling and the modified analytical solution (Eqs. [15] to [18]) for \( B = 3 \) m. One sees that the modified solution represents the vertical stress fairly well along the center-line (Figure 6b) and the horizontal stress everywhere (Figures 6a, 6c and 6e). An improvement is obtained with the modified formulation regarding the vertical stress along the wall (Figure 6d) and across the width of the stope (Figure 6f), compared to the original Marston-based solution. The same observation can be made for stopes with \( B = 6 \) m (Figure 7) and \( B = 18 \) m (Fig. 8). In all cases, the stresses are below those due to the overburden weight, indicating the occurrence of arching effects. These figures also show another Marston-based solution proposed by Drescher (1991), who assumed that both the horizontal and vertical stresses are non-uniformly distributed across the width. The additional complexity of the Drescher (1991) solution (not presented here) does not lead to an improved description of stresses in the stope.

![Figure 6: Calculation results showing the horizontal \( \sigma_h \) and vertical \( \sigma_v \) stresses for \( B = 3 \) m, \( \phi = 30^\circ \): (a) \( \sigma_h \) along the vertical center line; (b) \( \sigma_v \) along the vertical center line; (c) \( \sigma_h \) along the wall; (d) \( \sigma_v \) along the wall; (e) \( \sigma_h \) at different depths across the width; (f) \( \sigma_v \) at different depths across the width.](image)

3 DISCUSSION AND CONCLUSION

In this paper, an existing analytical solution has been modified to obtain the stress state in backfilled stopes under plane strain conditions. This solution can be used when the stope is long compared to its width. Otherwise, the effect of the third dimension should be taken into account (Li et al. 2005a). Li et al. (2005a) have also shown the effect of cohesion (neglected here) on the stress distribution (see also Li et al. 2007).
Figure 7: Calculation results showing the horizontal $\sigma_h$ and vertical $\sigma_v$ stresses for $B = 6$ m, $\phi = 30^\circ$: (a) $\sigma_h$ along the vertical center line; (b) $\sigma_v$ along the vertical center line; (c) $\sigma_h$ along the wall; (d) $\sigma_v$ along the wall; (e) $\sigma_h$ at different depths across the width; (f) $\sigma_v$ at different depths across the width.

The solution developed by the authors is based on the Coulomb yield criterion, which may not always be appropriate when dealing with backfill, particularly when tensile stresses or relatively high mean pressures are involved. In this regard, the authors have developed a general non-linear 3D criterion (e.g., Aubertin et al. 2000; Li et al. 2005b), which is considered more representative and is being used in complementary analyses.

Another important issue not included in this paper (but addressed elsewhere) is the effect of water on the response of backfill in stopes. Pore pressure build-up due to backfill addition, consolidation and limited drainage, evolution of cemented backfill properties during curing, and moisture retention in the backfill (under suction) are some of the complex (and coupled) processes that should be taken into account to obtain a more general picture of the behaviour of backfilled stopes in underground mines. Such aspects are being investigated by the authors and collaborators.

Despite its limitations, the modified analytical solution proposed here to evaluate earth pressures in narrow, vertical backfilled openings provides a practical means for the early stage of analysis. This solution includes a distribution factor, which can be calibrated using numerical modelling. The descriptive capability of the proposed solution compares well with numerical results. The proposed analytical solution can thus be used to estimate the earth pressure in narrow, vertical backfilled openings, including mining stopes and trenches.

ACKNOWLEDGEMENTS

The authors acknowledge the financial support from the participants of the Industrial NSERC Polytechnique-UQAT Chair on Environment and Mine Wastes Management (http://www.polymtl.ca/enviro-geremi/). The authors thank Dr. John Molson for his review of this manuscript.
Figure 8: Calculation results showing the horizontal $\sigma_h$ and vertical $\sigma_v$ stresses for $B = 18$ m, $\phi = 30^\circ$: (a) $\sigma_h$ along the vertical center line; (b) $\sigma_v$ along the vertical center line; (c) $\sigma_h$ along the wall; (d) $\sigma_v$ along the wall; (e) $\sigma_h$ at different depths across the width; (f) $\sigma_v$ at different depths across the width.

REFERENCES


