A CONSTITUTIVE MODEL TO PREDICT THE HYDROMECHANICAL BEHAVIOUR OF ROCK JOINTS

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ABSTRACT
The mechanical behaviour of rock joints can be strongly influenced by water flow, while the water pressure may in turn affect the joint response. These coupled phenomena should be considered simultaneously to simulate the behaviour of rock joints. The CSDS constitutive model was developed by the authors to represent the mechanical behaviour of dry rock joints. In this paper, the authors show how the CSDS model can be adapted to take into account the presence of water on the joint behaviour. The modified model formulation is presented and then validated with experimental data from hydromechanical tests taken from the literature. A discussion on the significance of the model follows.

RÉSUMÉ
Le comportement mécanique d’une discontinuité géologique peut être très influencé par l’écoulement de l’eau, alors que la pression de l’eau peut à son tour affecter le comportement du joint. Ces phénomènes couplés doivent être considérés simultanément pour simuler le comportement des discontinuités. Le modèle constitutif CSDS a été développé par les auteurs pour représenter le comportement mécanique des discontinuités dans des conditions sèches. Dans cet article, les auteurs montrent comment ce modèle peut être adapté pour prendre en compte la présence d’eau dans les discontinuités. Le formulation modifiée du modèle est présentée et ensuite validée à l’aide de résultats expérimentaux d’essais hydromécaniques tirés de la littérature. Une discussion sur l’approche proposée termine l’article.

1 INTRODUCTION
The mechanical response of rock masses is, in many situations, much more dependent on the properties of joints than of intact rock. For instance, the deformation and failure mechanisms of fractured rock masses are often governed by shear along discontinuities. A good understanding of joints behaviour is thus a basic prerequisite for a comprehensive description of the complex mechanical behaviour of the medium. A model for rock joints, called the CSDS model (for Complete Stress-Displacement Surface), has been developed by Simon (1999; see also Simon et al. 1999). This model provided a representative description of rock joints (shear and normal) behaviour in the pre-peak as well as in the post-peak phases under dry conditions.

A rock mass behaviour can also be influenced by the water flow and ensuing pore pressure. For example, a previously stable rock structure can become unstable with an increase of water pressure inside the joints. Thus, it is essential to be able to predict the hydromechanical behaviour of rock joints in the presence of water to better assess the stability of rock structures under many situations. The accurate prediction of fluid flow through rock joints and water interaction with existing excavations can be very important for environmental protection of water resources (e.g. Ben Abdelgahni et al. 2007). In this paper, the authors present a modification of the CSDS model to take the effect of water pressure into account. It is shown that this modified formulation can be effective in predicting the hydromechanical behaviour of joints under laboratory testing conditions.

2 THE CSDS MODEL FOR ROCK JOINTS
2.1 The shear stress - shear displacement relationship
The CSDS model has been developed to fully describe the behaviour of dry rock joints in pre-peak and post-peak phases (Simon 1999, Simon et al. 1999). It can be written as follows for the shear stress - shear displacement relation:

\[ \tau = a + b \exp(-c u) - d \exp(-e u) \]

where \( \tau \) is the shear stress (MPa), \( u \) is the shear displacement (mm), and \( a \) to \( e \) are model parameters which must satisfy the condition \( c < e \) \( (a, b, c, d, e > 0) \). These parameters can be determined from the following relationships:

\[ a = \tau_i \]
\[ b = d - a \]
\[ c = 5 / u_i \]

Parameters \( d \) and \( e \) can be obtained by solving the following equations:
\[
\frac{d}{d\tau_r} \left[ \frac{e u_r}{5(d - \tau_r)} - \exp \left[ \frac{u_p (e - \frac{5}{\tau_r})}{u_r} \right] \right] = 0 \tag{5}
\]

\[
\tau_p - \tau_r \left[ 1 - \exp \left( -\frac{5u_p}{u_r} \right) \right] \frac{d}{d\tau_r} \exp \left( -\frac{5u_p}{u_r} \right) - \exp \left( -\frac{5e u_p}{u_r} \right) \tag{6}
\]

In these equations, \( \tau_r \) is the residual strength, \( \tau_p \) is the peak strength, \( u_p \) is the displacement at peak strength and \( u_r \) is the displacement at the onset of \( \tau_r \). Equations 5 and 6 must be solved simultaneously to evaluate the values of parameters \( d \) and \( e \). The development and use of these equations can be found in Simon (1999) and Simon et al. (1999). The model can also predict rock joint behavior for determining the post-peak behavior of intact rock (Simon et al. 2003); additionally, it has been extended to take into account scale effects (Deng et al. 2004). In the following, the CSDS model is modified to treat the effect of water pressure.

2.2 The normal displacement - shear displacement relationship

An exponential formulation has also been used to describe the complementary normal displacement \( (v) \) to shear displacement \( (u) \) relation. This relation can be expressed as (Simon 1999, Simon et al. 1999):

\[
v = \beta_1 - \beta_2 \exp(-\beta_3 u) \tag{12}
\]

where \( \beta_1 \), \( \beta_2 \) and \( \beta_3 \) are model parameters. The value of these parameters is given by:

\[
\beta_1 = u_r \left( \frac{1}{\sigma_n} \right) \tan \frac{k_2}{\tan i_0} + \frac{\sigma_n V_m}{k_n V_m - \sigma_n} \tag{13}
\]

\[
\beta_2 = \beta_1 - \frac{\sigma_n V_m}{k_n V_m - \sigma_n} \tag{14}
\]

\[
\beta_3 = \frac{1.5}{u_r} \tag{15}
\]

where \( V_m \) is the maximum closure of the joint and \( k_n \) is the initial normal stiffness of the joint.

Figure 1 shows schematic curves obtained with the CSDS model. This model has been applied to several test data taken from the literature, and the results indicate that it can be representative of the shear stress - shear displacement behaviour of rock joints (Simon 1999, Simon et al. 1999). The model can also predict rock joint behaviour under constant normal stiffness conditions (Simon 1999, Simon et al. 1999). The CSDS model may also be used for determining the post-peak behaviour of intact rock (Simon et al. 2003); additionally, it has been extended to take into account scale effects (Deng et al. 2004). In the following, the CSDS model is modified to treat the effect of water pressure.

3 PREDICTION OF THE HYDROMECHANICAL BEHAVIOUR WITH THE CSDS MODEL

3.1 Mechanical behaviour

Tremblay (2005) used the CSDS model to analyze the hydromechanical behaviour of rock joints. Several studies have shown that the presence of water inside a discontinuity can reduce significantly its shear strength (e.g. Tsang 1990; Lamontagne 2001). The simplest explanation for this phenomenon relies on the notion of effective stress, developed and used in soil mechanics (Terzaghi 1936; Mitchell 1976). The same approach has also been used in rock mechanics (e.g. Skempton 1960; Mesri and Gibala 1972; Goodman 1980). When a fluid
pressure is acting in a rock joint, the effective normal stress ($\sigma_n'$) on the joint surface is given by:

$$\sigma_n' = \sigma_n - P_w$$  \[16\]

where $\sigma_n$ is the global normal stress on the rock joint and $P_w$ is the water pressure. The latter induces a reduction of normal stress, and thus leads to a lowering of the peak and residual shear strengths (given by Equations 7 and 8). The water pressure may also increase the dilatation of the joint (as given by Equation 12). In a rock joint, the influence of the fluid pressure may be seen as being similar to the influence of a reduced normal stress (which is illustrated in Figure 1). Here, it is considered that the presence of water does not modify the morphology of the joint nor its friction characteristics. The CSDS model is modified here by replacing $\sigma_n$ by $\sigma_n'$ in all the above equations.

Figure 1. Typical curves obtained with the CSDS model

3.2 Hydraulic behaviour

The rock joint mechanical behaviour when sheared will also have an influence on its hydraulic conductivity. Figure 2 shows the results obtained by Mainy (1971) on a dilatant joint under low normal stress. A shear displacement produces a normal displacement, which in turn increases the conductivity.

To model the water flow through a rock joint, many authors have used the analogy of two parallel and perfectly smooth plates. The analytical solution to the Navier-Stockes equations for a laminar fluid flow can then be used. This solution is expressed by (e.g. Witherspoon et al. 1981):

$$Q_f = V_f A_{sec} = - \frac{\rho g b \Delta w \Delta h}{12 \mu L}$$  \[17\]

with

$$A_{sec} = b \ w$$  \[18\]

where $Q_f$ is the flow in the rock joint (m³/s), $V_f$ is the mean velocity (m/s),$A_{sec}$ is the area perpendicular to the flow (m²), $b$ is the distance between the rock joint surfaces or joint hydraulic opening (m), $w$ is the joint dimension perpendicular to the flow (m), $L$ is the path length parallel to the flow (m),$\Delta h$ is the hydraulic gradient (m), $g$ is the gravitational acceleration (m/s²), $\rho$ is the water density (kg/m³) and $\mu$ its dynamic viscosity (kg/ms).

The intrinsic permeability of a joint $k_f$ (m²) and intrinsic transmissivity $T_f$ (m³) (which are independent of the type of fluid) are then given by:

$$k_f = \frac{b^2}{12}$$  \[19\]

$$T_f = \frac{b^3}{12}$$  \[20\]

Since a natural rock joint is far from being formed by two parallel smooth plates, the mechanical opening does not correspond to the hydraulic opening, although they are related. Several authors have proposed empirical equations to relate the mechanical and hydraulic openings (e.g. Barton 1982; Barton et al. 1985; Cook 1988; Zhang and Sanderson 2002).

As the hydraulic opening is proportional to the mechanical opening, it can be postulated that a mechanical opening increase (or decrease) caused by shearing of the joint will cause an equal increase (or decrease) in the hydraulic opening. Hence, the variation of the hydraulic opening $\Delta b$ caused by shearing can be expressed by:

$$\Delta b = \Delta v$$  \[21\]
where $\Delta v$ is the variation of the mechanical opening from its initial state. The intrinsic transmissivity of a rock joint when sheared can then be calculated with the CSDS model by combining Equations 12 and 20:

$$T_f(u) = \left(\frac{b + \Delta v}{12}\right)^3 \quad [21]$$

with

$$\Delta v = \beta_2 \left[1 - \exp\left(-\beta_3 u\right)\right] \quad [22]$$

To use this approach, the initial flow properties of the rock joint must be known so the value of the initial hydraulic opening $b$ can be established.

4 APPLICATIONS

To illustrate how the modified CSDS model can be used to describe, and in some cases predict the mechanical behaviour of rock joints under hydraulic pressure, different experimental results taken from the literature have been analysed. Some of the main results are presented here; more details are given in Tremblay (2005).

4.1 Data taken from Olsson (1998)

Olsson (1998) tested a granitic rock joint replica made from high strength mortar. The uniaxial compressive strength of the mortar was 200 MPa. Several shear tests were performed under an initial normal stress of either 2 or 4 MPa, with three constant normal stiffness $K = 0$ MPa/mm (i.e. constant normal stress), $K = 1.2$ MPa/mm and $K = 2.4$ MPa/mm.

The tests were first performed without fluid, and then an undrained fluid pressure of 0.04 MPa was imposed. Figure 3 shows the comparison between the CSDS model curve and the test results for the dry conditions. The model parameters (given in Table 1) were obtained from the constant normal stress tests. The curves showed in Figure 3b are the model predictions based on that initial calibration. As it can be seen, the CSDS model can reproduce fairly well the mechanical behaviour of that joint without water pressure.

Figure 4 shows similar comparisons between the predicted curves from the CSDS model and the tests results with a water pressure of 0.04 MPa. Here again, the prediction made with the CSDS model can be representative of the actual joint response.
4.2 Data taken from Gentier et al. (1997)

Gentier et al. (1997) performed a series of hydromechanical shear tests on a granitic rock joint replica made from mortar under a constant normal stress of 7 MPa in three different directions. The uniaxial compressive strength of the mortar was 82 MPa. In their experiment, they studied the variation of flow and permeability when the joint was sheared. The water pressure in these tests was 0.072 MPa.

Figure 5 shows the comparison between the CSDS model curves and the test results (the model parameters values are given in Table 2). Here again, the predicted behaviour of the joint with the modified CSDS model compares fairly well with the test results. It should be mentioned here that the water pressure was not always held constant during shearing in these tests, which may have cause the relatively large dispersion of the data.

Table 1. CSDS model parameters values used in the calculations shown in Figures 3 and 4

<table>
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<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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</thead>
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<td>( \theta ) [degree]</td>
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Table 2. CSDS model parameters values used in the calculations shown in Figure 5

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<th>Test 5</th>
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<td>( V_m ) [mm]</td>
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<td>-110</td>
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<td>( \phi_o ) [degree]</td>
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<tr>
<td>( S_o ) [MPa]</td>
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4.3 Data taken from Lamontagne (2001)

Lamontagne (2001) performed hydromechanical shear tests on a granitic rock joint replica made from mortar under a constant normal stress. The uniaxial compressive strength of the mortar was 74 MPa. The tests were first performed in a dry condition, and then the same replica was tested with a water pressure of 0.16 MPa. Figure 6 shows the comparison between the CSDS model and the dry tests results (the model parameters values are given...
in Table 3); Figure 7 shows the predicted curves for the hydromechanical tests.

![Graph](image)

Figure 5. Results calculated with the CSDS model and obtained from shear tests with a water pressure of 0.072 MPa under a constant normal stress of 7 MPa (experimental data taken from Gentier et al. 1997, after Tremblay 2005).

<table>
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<th>Parameter</th>
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<th>Parameter</th>
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<td>$S_o$ [MPa]</td>
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</table>

Table 3. CSDS model parameters values used in the calculations shown in Figures 6 and 7

The model parameters (given in Table 3) were calibrated based on the dry tests. The curves obtained in Figure 7 are the model predictions based on this calibration. Here again, the predictions obtained from the CSDS model are very good.

Lamontagne (2001) also measured the intrinsic transmissivity during the shear tests. Figure 8 shows the variation of the intrinsic transmissivity predicted with the CSDS model (Equation 21) and the experimental results measured during the tests. In this figure, the value of the hydraulic opening $b$ was back calculated from the transmissivity value at $u = 0$ with Equation 20. Then, Equation 21 was used to calculate the predicted value.

It can be seen that the correlation between the predicted values and the experimental data is fairly good up to a shear displacement of 1 mm (which is twice the peak displacement). After that point, the model tends to over estimate the value of $T_r$, by up to an order of magnitude. This discrepancy can be explained by the fact that once the peak strength has been attained, the sheared asperities may cause an obstruction that increases the head loss and reduces the flow rate through the joint (and thus its transmissivity). This aspect is not taken into account in the model formulation.

![Graph](image)

Figure 6. Results calculated with the CSDS model and obtained from shear tests without water pressure under constant normal stress (experimental data taken from Lamontagne 2001, after Tremblay 2005).
Figure 7. Results calculated with the CSDS model and obtained from shear tests with a fluid pressure of 0.16 MPa under a constant normal stress (experimental data taken from Lamontagne 2001, after Tremblay 2005).

5 DISCUSSION AND CONCLUSION

The results presented above show that the CSDS model can provide a good description of rock joints behaviour under monotonic loading in either dry conditions or with a water pressure. The different applications presented here confirm that using the concept of effective stress leads to a good representation of the experimental data taken from the literature.

In the approach proposed here, it was considered that the water pressure did not create any modification to the geometry of the rock joint. In other words, the fluid pressure is not high enough to create a separation of the joint walls or even to increase the initial opening. When the fluid pressure is high compared to the acting normal stress, the contact between the wall surfaces will be reduced and the mechanical behaviour may be changed significantly. This may be taken into account with the CSDS model by considering the fluid as joint filling. Deng et al. (2006) proposed an effective approach to represent the presence of dry filling inside rock joints with the CSDS model. This aspect will be addressed in future publications.

The presence of a fluid inside the rock joint can also cause a reduction of the frictional stress along joint walls. Barton and Choubey (1977) have shown that the friction angle can be reduced under wet conditions. This aspect was also neglected in the applications presented here, but could be taken into account with the CSDS model if the values of both the wet and dry friction angles are known.

By using the normal behaviour calculated with the CSDS model, it is possible to assess the evolution of fluid flow inside the rock joint. However, because the shearing of asperities inside the joint may cause obstructions and increase the head loss inside the joint, these predictions can only be accurate until asperity shearing becomes significant (i.e. beyond peak strength). For the same reason, the predictions are more accurate at low normal stresses as a lesser proportion of asperities are sheared.

Despite its limitations, this paper has shown that the modified CSDS model can be used to assess the effect of water pressure on the response of rock joints under a variety of situations.

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